Economic Ports for Optimal Operation of Networked AC Microgrids

Okonomische Ports für die Optimale Regelung von vernetzte AC Microgrids

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Abstract — In this paper, we propose the recently introduced *economic ports* for AC microgrids, allowing a modular and optimal operation of networked microgrids. Firstly, we design a price-based controller for optimal operation of a single microgrid. Secondly, we design economic and electric interconnection ports for networked microgrid operation. Lastly, we propose an interconnection scheme for the economic ports. This interconnection scheme drives the networked microgrids to an optimal setpoint in a distributed manner, only requiring an exchange of local prices with neighbored microgrids. The methodology is verified through representative simulation studies.

Zusammenfassung — In diesem Paper werden die neuartigen, sog. ökonomische Ports (engl. *economic ports*), die kürzlich für DC Microgrids vorgeschlagen wurden, für AC-Systeme vorgestellt. Als Erstes wird eine preisbasierte Regelung für ein einzelnes Microgrid entworfen, welche einen lokalen Preis für einen Microgrid definiert. Danach werden ökonomische und elektrische Ports für AC Microgrids definiert. Schließlich wird eine Methode vorgeschlagen, mit denen die ökonomische Ports vernetzt werden können und zu globaler Optimalität der vernetzten Microgrids führt. Dies wird anschließend an akademischen Beispielen verdeutlicht und verifiziert.

I. INTRODUCTION

Power systems are undergoing a transformation towards a sustainable and emission-free electrical supply based on renewable energies. Due to the integration of volatile renewable energy sources and the removal of large-scale fossil fuel generators, power systems are experiencing a reduction in grid inertia and are hence increasingly facing grid stability issues. With the ongoing removal of large-scale generators, the distributed generation units (DGUs) need to contribute to grid stability. Due to the large number of small-scale DGUs in an energy system, the optimal coordination of the DGUs and the distributed operation are crucial.

In Literature, many approaches propose a passivitybased controller design for DGUs in AC microgrids [1, 2]. These regulators achieve an offset-free regulation of a given voltage reference and have desirable plug-and-play properties while guaranteeing asymptotic stability of the overall interconnected system via passivity. Although allowing plug-and-play operation and ensuring asymptotic stability, passivity-based methods are in general purely decentralized approaches which cannot achieve an economically optimal operation or steer the system to an economically optimal steady state.

Addressing this issue, [3, 4, 5] propose distributed passivity- and optimization-based controllers for a microgrid in port-Hamiltonian form that is able to steer the system to an economically optimal steady state. The intrinsic, favorable passivity properties of the port-Hamiltonian system enable plug-and-play operation while ensuring asymptotic stability. However, in both approaches, the whole microgrid is modeled as a synchronous generator, which is interconnected with other microgrids via lossless, static lines. These simplifications and assumptions, although allowing important theoretical contributions, hamper the application to low inertia microgrids with lossy lines, which will adopt a crucial role in future power systems. Another line of research proposes to use model predictive control (MPC) for microgrids [6, 7, 8]. In [8], a model predictive controller exploiting the passivity properties of the underlying controllers for ensuring modular stability is proposed. This method achieves an optimal operation, but requires considerable amount of computing power each time step and does not allow a distributed operation. In [9], an optimization based controller for AC microgrids with underlying droop controllers is proposed. Although these methods are not based on passivity, asymptotic stability of an economically optimal steady state together with plug-and-play capabilities are shown. However, this again comes at the cost of considering a system model with limiting assumptions and approximations, e.g. static lines and single capacitance dynamics as microgrid model or a simple oscillator as AC microgrid node dynamics. In particular, dynamics of the DGUs, transmission lines or nonlinear loads are not considered.

Recently, the concept of *economic ports* has been introduced in the context of DC microgrids [10]. These allow a distributed economically optimal operation of DC microgrids while allowing a modular stability analysis by analyzing the dissipativity properties of these ports.

Contributions: In this work, we introduce the novel economic ports for AC microgrids. Similar as in the DC case [10], each microgrid has a local price for the power infeed defined by the stabilizing action of a grid-forming DGU, which has the only task of stabilizing the microgrid. For achieving networked optimal operation, we define economic ports in the context of AC microgrids and use a distributed consensus algorithm in order to achieve economic operation of the whole, networked microgrid in a distributed manner.

The reminder of this paper is structured as follows. In Section 2, the system model is presented. In Section 3, the price-based controller for a single microgrid is presented. The economic ports for an interconnection with other microgrids is described in Section 4. Lastly, we present simulation results to verify our control design.

II. SYSTEM MODEL

In this paper, we consider a set of microgrids $k \in \mathcal{M} = \{1, \dots, n_{\text{mg}}\}$, each comprising a set $\mathcal{B}^k$ of $n^k = |\mathcal{B}^k|$ electrical buses or nodes connected via a set $\mathcal{E}^k$ of $m^k = |\mathcal{E}^k|$ electrical lines. Defining an arbitrary line current direction over the microgrid power lines, we describe the network topology of each microgrid with the directed graph $\mathcal{G}^k(\mathcal{B}^k, \mathcal{E}^k)$, where $\mathcal{B}^k$ is the set of nodes and $\mathcal{E}^k$ of edges. In the following, we present the dynamic models of the AC microgrid components. We consider nodes $i \in \mathcal{B}_{\text{L},k} \subseteq \mathcal{B}^k$ having only a nonlinear load, and nodes $i \in \mathcal{B}_{\text{DGU},k} \subseteq \mathcal{B}^k$ having additionally a DGU, with $\mathcal{B}_{\text{L},k} \cup \mathcal{B}_{\text{DGU},k} = \mathcal{B}^k$. When equipped with a DGU, the node voltage v_i can be directly influenced.

In the remainder of this section, the dynamic models of the microgrid components are presented. The microgrid index k (always displayed as superscript) is omitted for simplicity until further notice, since the same microgrid structure holds for all $k \in \mathcal{M}$ (microgrids may have different sizes, topologies and parameters).

To avoid dealing with sinusoidal signals in the control design process, electrical signals in this study are expressed within a dq reference frame (see, e.g. [11]) that is shared among all DGUs operating at a common speed, denoted as ω_0. Therefore the dq reference frame variables are denoted by

$$x^{dq} = x^d + \mathrm{i}x^q, \tag{1}$$

where x represents either a current or a voltage in the dq reference frame and i denotes the imaginary part.

A. *Distributed generation unit (DGU)*

A DGU node $i \in \mathcal{B}_{\text{DGU}}$ consists of a direct current (DC) voltage source, typically derived from a renewable source, a voltage source converter (VSC) supplying a voltage $v_{\text{t},i}^{dq}$, and a filter with inductance $L_{\text{f},i} \in \mathbb{R}_{>0}$, capacitance $C_{\text{f},i} \in \mathbb{R}_{>0}$, and resistance $R_{\text{f},i} \in \mathbb{R}_{>0}$. The VSC and the filter form a DGU (see Figure 1). Each bus $i \in \mathcal{B}$ has two states, the node voltage and the filter current. The VSC voltage $v_{\text{t},i}^{dq}$ is defined as the system input. On applying Kirchhoff's current law and Kirchoff's voltage law on the DGU side [1], we obtain a DGU model in the dq reference frame as (rotating with speed ω_0)

$$\frac{\mathrm{d}}{\mathrm{d}t} v_i^{dq} = -\mathrm{i}\omega_0 v_i^{dq} + \frac{i_{\text{f},i}^{dq}}{C_{\text{f},i}} - \frac{i_{\text{ext},i}^{dq}}{C_{\text{f},i}} \tag{2a}$$

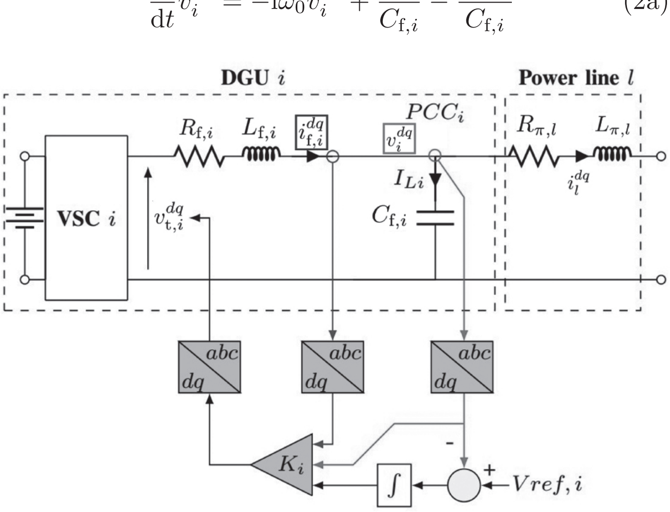


Fig. 1. Electrical scheme of DGU i, power line l, and local PnP voltage and frequency controller[1]

$$\frac{\mathrm{d}}{\mathrm{d}t} i_{\text{f},i}^{dq} = -\left(\frac{R_{\text{f},i}}{L_{\text{f},i}} + \mathrm{i}\omega_0\right) i_{\text{f},i}^{dq} - \frac{v_i^{dq}}{L_{\text{f},i}} + \frac{v_{\text{t},i}^{dq}}{L_{\text{f},i}}, \tag{2b}$$

where $i_{\text{f},i}^{dq}$ represents the filter current and v_i^{dq} the node voltage of the i-th DGU. In (2), $i_{\text{ext},i}^{dq}$ is defined as the cumulative current injected by interconnecting lines.

B. *Grid-forming and grid-following controllers*

The DGU described in Section II-A is normally equipped with a grid-forming or a grid-following controller [10]. Grid-forming controllers inject the necessary filter current $i_{\text{f},i}^{dq}$ (indirectly power) in order to regulate the node voltage v_i^{dq} to a desired voltage reference v_{ref}^{dq}, and stabilize thus the grid-voltages regardless of the load disturbance or volatile power injections. Grid-following controllers set the voltage $v_{\text{t},i}^{dq}$ such that a given power reference $p_{\text{ref}}/q_{\text{ref}}$ (indirectly the filter current $i_{\text{f},i}^{dq}$) is injected, without considering the resulting node voltage level v_i^{dq} .

Grid-forming DGUs are used to achieve robust voltage stability, and grid-following DGUs to inject a certain amount of power irrespective of grid stability, e.g. for achieving optimal dispatch. Therefore, in this work, we have exactly one grid-forming DGU in every microgrid, and an arbitrary number of grid-following DGUs. The first DGU $1 \in \mathcal{B}_{\text{DGU}}$ is defined without loss of generalization as the grid-forming DGU, while $i \in \mathcal{B}_{\text{DGU}} \setminus \{1\}$ are grid following DGUs. The grid-forming controller is taken from [1] and is designed by introducing an error state (3b) and a state feedback as

$$v_{\text{t},1}^{dq} = k_{\alpha,1} v_1^{dq} + k_{\beta,1} i_{\text{f},1}^{dq} + k_{\gamma,1} e_1 \tag{3a}$$

$$\dot{e_1} = v_{\text{ref}}^{dq} - v_1^{dq}, \tag{3b}$$

where $k_{\alpha,1} \in \mathbb{R}$, $k_{\beta,1} \in \mathbb{R}$ and $k_{\gamma,1} \in \mathbb{R}$ are the control parameters. For designing the grid-following controller, active and reactive power p and q are defined as [12]

$$p = v^d i^d + v^q i^q \tag{4a}$$

$$q = v^d i^q - v^q i^d. \tag{4b}$$

Using a similar formula as in the grid-forming controller, the grid-following controller is designed here as

$$v_{\text{t},i}^{dq} = k_{\alpha,i} v_i^{dq} + k_{\beta,i} i_{\text{f},i}^{dq} + k_{\gamma,i} e_i \tag{5a}$$

$$\dot{e_i} = p_{\text{ref}} - (v_i^d i_{\text{f},i}^d + v_i^q i_{\text{f},i}^q) + \mathrm{i}(q_{\text{ref}} - (v_i^d i_{\text{f},i}^q - v_i^q i_{\text{f},i}^d)), \tag{5b}$$

for all $i \in \mathcal{B}_{\text{DGU}} \setminus \{1\}$, using the injected power error (5b) instead. Note that the grid-following DGU introduces a nonlinearity when computing the injected power p_i or q_i in (5b). The active power (p_{ref}/p_i) and reactive power (q_{ref}/q_i) are combined within s_{ref}/s_i, where the active power represents the real component and the reactive power represents the imaginary component. The power is determined using the node voltage v_i^{dq} and the filter current $i_{\text{f},i}^{dq}$. Applying either (3) or (5) to the DGU (2) thus yields

$$\frac{\mathrm{d}}{\mathrm{d}t} v_i^{dq} = -\mathrm{i}\omega_0 v_i^{dq} + \frac{i_{\text{f},i}^{dq}}{C_{\text{f},i}} - \frac{i_{\text{ext},i}^{dq}}{C_{\text{f},i}} \tag{6a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} i_{\text{f},i}^{dq} = \alpha_i v_i^{dq} + \beta_i i_{\text{f},i}^{dq} + \gamma_i e_i \tag{6b}$$

$$(3\text{b}) \text{ or } (5\text{b}), \tag{6c}$$

depending if it is a grid-forming or grid-following DGU, with the variables $\alpha_i = \frac{k_{\alpha,i}-1}{L_{\text{f},i}}$, $\frac{k_{\beta,i}-R_{\text{f},i}}{L_{\text{f},i}}$ and $\gamma_i = \frac{k_{\gamma,i}}{L_{\text{f},i}}$ containing the controller parameters. The electrical circuit including the internal voltage and frequency controller of a grid-forming DGU are shown in 1.

Note that α_i, β_i and γ_i are real values. Therefore $v^d_{\mathrm{t},i}$ (and indirectly $i^d_{\mathrm{f},i}$) depends only on the d-components of v^{dq}_i, $i^{dq}_{\mathrm{f},i}$ and e_i. Similarly $v^q_{\mathrm{t},i}$ (and indirectly $i^q_{\mathrm{f},i}$) depends only on the q-components of v^{dq}_i, $i^{dq}_{\mathrm{f},i}$ and e_i. In this case, the dq components of the grid-forming and grid-following controllers are decoupled. This brings along some advantages, which are discussed in [13].

C. *Power lines*

The π-model of the power lines $l \in \mathcal{E}$ is taken from [1] and [10] and is shown in figure 1 or 2. Note that the two capacities $\frac{C_{\pi,l}}{2}$ from the π-model can be neglected, as they are lumped with the parallel node capacitance $C_{\mathrm{f},i}$ or $C_{\mathrm{L},j}$. In this case, the power line is modeled as a combination of resistance $R_{\pi,l}$ and inductance $L_{\pi,l}$. Using the dynamic model from [1], one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t} i^{dq}_l = -(\frac{R_{\pi,l}}{L_{\pi,l}} + \mathrm{i}\omega_0) i^{dq}_l + \frac{\Delta v^{dq}_l}{L_{\pi,l}} \tag{7}$$

where Δv^{dq}_l defines the voltage drop over the transmission line. It can be interpreted as the voltage difference between the two nodes connected by the -lth transmission line.

D. *Load node*

A load node $j \in \mathcal{B}_\mathrm{L}$ is composed of a capacitance together with a nonlinear load. The load, in this case ZIPload, consists of four components connected in parallel. A capacitance $C_{\mathrm{L},j}$, a constant impedance (Z), consisting of a resistance $\frac{1}{Y_{\mathrm{L},j}}$, a constant current load (I) $i^{dq}_{\mathrm{LI},j}$ and a constant power load (P). The dynamic model in the dqframe with the constant admittance and constant current load (ZI-load) is taken from [1].

The constant power load was modeled as a second current load $i^{dq}_{\mathrm{LP},j}$ with variable current. In this case $p_{\mathrm{L},j}$ and

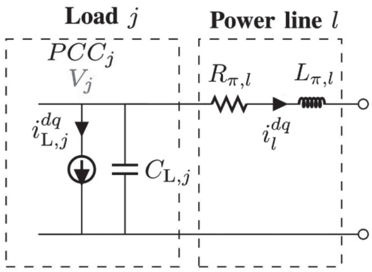


Fig. 2. Electrical scheme of ZIP load and the power line. The current $i^{dq}_{\mathrm{L},j}$ incorporates the effects of all three load types[1]

$q_{\mathrm{L},j}$ are given by the system. The node voltage v^{dq}_j is given by the grid-forming DGU and all other network components and parameters. Therefore the two values to be determined are the dq-components of $i^{dq}_{\mathrm{LP},j}$. They are defined as

$$i^q_{\mathrm{LP},j} = \frac{q_{\mathrm{L},j} v^d_j + p_{\mathrm{L},j} v^q_j}{(v^d_j)^2 + (v^q_j)^2} \tag{8a}$$

$$i^d_{\mathrm{LP},j} = \frac{p_{\mathrm{L},j} v^d_j - q_{\mathrm{L},j} v^q_j}{(v^d_j)^2 + (v^q_j)^2}. \tag{8b}$$

The complete load node model is shown in (8a)

$$\frac{\mathrm{d}}{\mathrm{d}t} v^{dq}_j = -\mathrm{i}\omega_0 v^{dq}_j - \frac{i^{dq}_{\mathrm{L},j}}{C_{\mathrm{L},j}} - \frac{i^{dq}_{\mathrm{ext},i}}{C_{\mathrm{L},j}} \tag{9a}$$

$$\rightarrow i^{dq}_{\mathrm{L},j} = Y_{\mathrm{L},j} v^{dq}_j + i^{dq}_{\mathrm{LI},j} + i^{dq}_{\mathrm{LP},j}. \tag{9b}$$

The total load current $i^{dq}_{\mathrm{L},j}$ consists of the Z, I and P currents as shown in (8b). The cumulative current injected by interconnecting lines $i^{dq}_{\mathrm{ext},i}$ is defined as in (2). The electric scheme for the load node and the power line is shown in Figure 2.

In the last section, all the microgrid components together with their differential equations have been presented. In the following, we present the controllers for the grid following DGUs, that are intended to steer the system to an optimal steady state.

III. Controller Design

In this Section, we propose a controller design that (i) steers the microgrid to an unknown optimal steady state, that depends on the loads and network parameters, and (ii) that has a distributed nature without requiring a central unit.

Inspired by the Linear Convex Optimal Steady-State Control [14], we introduce an optimality model, which describes an optimal steady state where property (i) is fulfilled:

$$\min_{p_\mathrm{ref}} \sum_{i=1}^{d-1} f_i(p_{\mathrm{ref},i}) \tag{10a}$$

$$\text{s.t.} \sum_{i=1}^{d-1} p_{\mathrm{ref},i} = p_\mathrm{L}. \tag{10b}$$

The function $f_i : \mathbb{R} \rightarrow \mathbb{R}$ represents the cost of the active power infeed of the respective grid-following DGUs, which is assumed to be convex and quadratic in the paper at hand, i.e. $f(p_{\mathrm{ref},i}) = \nu_i p^2_{\mathrm{ref},i} + \rho_i p_{\mathrm{ref},i} + \mu_i$ with $\nu_i, \rho_i, \mu_i \in \mathbb{R}$, $\nu_i > 0$. The variable p_L comprises the sum of the power consumed by all loads and the losses of the microgrid. Thus, (10b) ensures power balance. Note that the reactive power q is not specified in the optimality model (10). In this work, we assume that the necessary reactive power is provided by the grid-forming DGU. The KKT conditions [15] for (10) are

$$0 = \nabla f_i(p_\mathrm{ref}) + \lambda \qquad \forall i \in \{1, \ldots, d-1\} \tag{11a}$$

$$0 = \sum_{i=1}^{d-1} p_{\mathrm{ref},i} - p_\mathrm{L}, \tag{11b}$$

where $\lambda \in \mathbb{R}$ is the Lagrange multiplier for the constraint (10b), and using a primal-dual gradient method [16] with positive tuning parameters τ_i and κ, we get

$$\dot{p}_{\mathrm{ref},i} = -\tau_i(\nabla f_i(p_{\mathrm{ref},i}) - \lambda) \quad \forall i \in \{1, \ldots, d-1\} \tag{12a}$$

$$\dot{\lambda} = \kappa(p_\mathrm{L} - \sum_{i=1}^{d-1} p_{\mathrm{ref},i}). \tag{12b}$$

The multiplier λ can be interpreted as the electrical power price; if the load p_L is greater than the power supplied by the grid-following DGUs, the price in (12b) increases and vice-versa. Equation (12a) means that the grid-following DGUs inject power such that their marginal costs equal the power price. This is the best solution for rational decisionmakers, since feeding in more power would lead to less economic benefit per kW.

With controller (12), property (i) is fulfilled, since at steady state, every grid-following DGU produces at marginal cost and the grid-forming DGUs inject no power. However, property (ii) requires more attention. Even if (12a) can be computed by every grid-following DGU in distributed manner, the price-forming (12b) uses the load p_L and the sum of the power injection $p_{\mathrm{ref},i}$ of the grid following DGUs, both being system-wide knowledge. To circumvent that, we use instead the power injected by the grid-forming DGU as follows

$$\dot{p}_{\mathrm{ref},i} = -\tau_i(\nabla f_i(p_\mathrm{ref}) - \lambda) \qquad \forall i \in \{1,\dots,d-1\} \quad (13a)$$
$$\dot{\lambda} = -\kappa y. \quad (13b)$$

The power injected by the grid-forming DGU is here as a measure for the unmet power demand in the microgrid. Since the grid-forming DGU is voltage controlled, it injects power whenever its voltage error is not zero. Thus, if the power injected by the grid-forming DGU is zero, it holds that (i) the grid-following DGUs inject power such that all loads and losses are met, and (ii) the voltage error at the DGU is zero. That way, with (13) we achieve the same steady state as with (12), but in a distributed manner and without necessitating knowledge about all loads.

For the sake of simplicity, we represent the closed-loop system defining the state variable $x = \mathrm{col}\{v, i_\mathrm{f}, e, i_\pi, p_\mathrm{ref}, \lambda\} \in \mathbb{R}^{n_\mathrm{cl}}$ with $n_\mathrm{cl} = n + 3d + m$ the number of states. Let $\bar{x}$ be an equilibrium point of the closed-loop system for a constant v_ref . In shifted coordinates $\tilde{x} = x - \bar{x}$, the nonlinear input-affine closed-loop system reads

$$\dot{\tilde{x}} = f(\tilde{x}, \bar{x}, P)\tilde{x} + Bp_\mathrm{ref}.$$

IV. INTERCONNECTION OF MICROGRIDS

In this section, we study the interconnection of various microgrids. First, we define the electric and novel economic ports for interconnecting with other microgrids $k \in \mathcal{M}$. Thereafter, we propose a interconnection scheme for the economic ports in order to achieve global optimal operation.

A. *Interconnection ports*

The following electric port defines an interface for interconnecting microgrids via electric lines.

Definition 1 (electric ports) *Let $i^k_{elec,i}$ be an external current injected at a node $i \in \mathcal{B}$ and v^k_i the voltage at that node for microgrid $k \in \mathcal{M}$. The input-output pair ($i^k_{elec,i}, v^k_i$) is called an electric port1 for that microgrid.*

The electric port is interfaced with system (14) through the vectors $b^k_\mathrm{elec} = \mathrm{col}\{t_i, 0_{3d+m}\}$ and $c^k_\mathrm{elec} = b^{k\top}_\mathrm{elec}$, where $t_i \in \mathbb{R}^n$ has a 1 at the i-th element and zero elsewhere, since an external current drawn to a node $i \in \mathcal{B}$ acts on the voltage dynamics (2a) of node $i \in \mathcal{B}$. Note that a microgrid may contain an arbitrary number $z \in \mathbb{R}$ of electric ports, yielding matrices $B^k_\mathrm{elec} = [b^k_{\mathrm{elec},1}, \dots, b^k_{\mathrm{elec},z}]$ and $C^k_\mathrm{elec} = B^{k\top}_\mathrm{elec}$.

The following economic port defines an interface for interconnecting microgrids economically.

Definition 2 (economic ports) *Let $\lambda^k_{ext} \in \mathbb{R}$ denote an external electric power price and $\lambda^k_{loc} \in \mathbb{R}$ the local price for a certain microgrid. The input-output pair ($\lambda^k_{ext}, \lambda^k_{loc}$) is called the economic port for microgrid $k \in \mathcal{M}$.*

When the economic port ($\lambda^k_\mathrm{ext}, \lambda^k_\mathrm{loc}$) is connected, we replace the price used for the grid-following DGUs in (13a) with the input λ^k_ext, yielding

$$\dot{p}^k_{\mathrm{ref},i} = -\tau^k_i(\nabla f^k_i(p^k_\mathrm{ref}) - \lambda^k_\mathrm{ext}) \quad (15a)$$
$$\dot{\lambda}^k_\mathrm{loc} = -\kappa^k y^k. \quad (15b)$$

The local price λ^k_loc (output of economic port) is still determined by the power injected by the grid-forming DGU, but s no longer used directly in the local microgrid. Splitting the price in a microgrid into local and external prices allows, using a special interconnection structure for economic ports as proposed in Section **??**, the local price λ^k_loc to contribute towards a (global) external price. The external price then already implicitly contains a coordination between microgrids, and is used by the grid-following DGUs in order to achieve global optimal dispatch. Note that only a single economic port per microgrid is allowed in this work, since we have a single local price per microgrid.

The economic port thus interfaces with the system (14) through the vectors $b^k_\mathrm{econ} = \mathrm{col}\{0_{n+2d+m}, \tau, 0\}$ and $c^k_\mathrm{econ} = \mathrm{col}\{0_{n+3d+m-1}, 1\}$. System (14) with electric and economic ports reads then

$$\dot{\tilde{x}}^k = f^k(\tilde{x}^k, \bar{x}^k, P^k)\tilde{x}^k + B^k_\mathrm{elec} i^k_\mathrm{elec} + b^k_\mathrm{econ}\lambda^k_\mathrm{ext} \quad (16a)$$
$$\tilde{y}^k_\mathrm{elec} = C^k_\mathrm{elec}\tilde{x}^k = v^k_\mathrm{elec} \quad (16b)$$
$$\tilde{y}^k_\mathrm{econ} = c^k_\mathrm{econ}\tilde{x}^k = \lambda^k_\mathrm{loc}. \quad (16c)$$

Vector $i^k_\mathrm{elec} = \mathrm{col}\{i^k_{\mathrm{elec},z}\}$ is the input for all electric ports z.

B. *Distributed consensus of local prices*

The goal of the interconnection scheme for the economic ports is to achieve equal prices in all microgrids. Then, we have global optimality, since the DGUs, which have individual costs for power injection, inject powe according to a single, global price. To achieve $\lambda^k_\mathrm{ext} = \lambda^l_\mathrm{ext}\ \forall k, l \in \mathcal{M}$, we propose a consensus-based algorithm with which the microgrids perform a distributed dynamic averaging of the local price λ^k_loc (output of the economic port). The output of the distributed dynamic averaging is used as the external price λ^k_ext (input of the economic port). Then, at steady state, the external prices λ^k_ext of all n_mg microgrids taking part in the distributed dynamic averaging are equal, i.e. $\lambda^k_\mathrm{ext} = \frac{1}{n_\mathrm{mg}}\sum_{k=1}^{n_\mathrm{mg}} \lambda^k_\mathrm{loc}$. There exist many dynamic consensus algorithms, see [17] for a survey. We choose the proportional dynamic consensus [17]

$$\dot{w} = -(\mu I_{n_\mathrm{mg}} + L)w - L\lambda_\mathrm{loc} \quad (17a)$$
$$\lambda_\mathrm{ext} = w + \lambda_\mathrm{loc}, \quad (17b)$$

where $L \in \mathbb{R}^{n_\mathrm{mg} \times n_\mathrm{mg}}$ is the Laplacian matrix of an arbitrary but connected topology describing the communication between the microgrids via economic ports, $\mu \in \mathbb{R}_{>0}$ a tuning parameter and $w \in \mathbb{R}^{n_\mathrm{mg}}$ auxiliary states. Note that the input $\lambda_\mathrm{loc} = \mathrm{col}\{\lambda^k_\mathrm{loc}\}$ and output $\lambda_\mathrm{ext} = \mathrm{col}\{\lambda^k_\mathrm{ext}\}$ of consensus algorithm (17) correspond to the economic port as described. All local prices thus contribute to the global, external price. This consensus protocol is chosen because it exhibits an excess of input and output passivity (it is input-to-state stable [18, Theorem 3] and has feedthrough, and is thus stabilizing).

[1] Note that electric ports have been used in the literature for interconnecting DGUs and lines [1] *within* a microgrid. Definition 1 can hence be understood as leveraging these ports *between* microgrids.

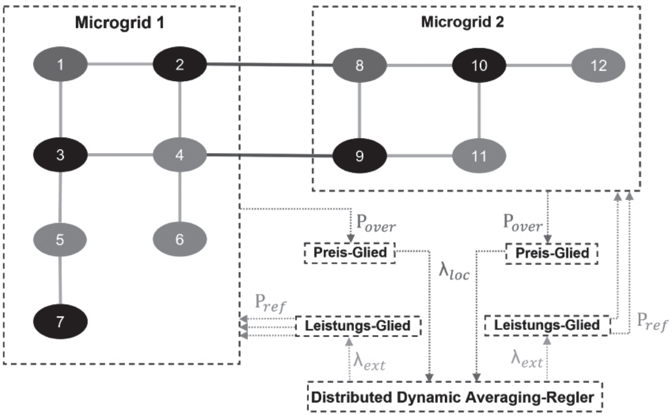
V. Simulation Results

In this section, we demonstrate the proposed controller design through an academic example. We show that the proposed communication scheme for the economic ports achieves global optimality. For the simulations, the software Matlab/Simulink is used.

We consider two interconnected microgrids as shown in Figure 3. Microgrid 1 is composed of 7 nodes and has 3 DGUs. Microgrid 2 is composed of 5 nodes and has 3 DGUs. The blue nodes contain a grid-forming DGU, green a grid-following DGU and black just a nonlinear load. In addition, the structure of the price forming mechanism and the consensus algorithm are shown. The microgrids are interconnected with 2 electric power lines. At times $t = [60, 90, 120, 150, 210, 250, 280, 300]$, load steps occur arbitrarily in one of both microgrids only. The load steps occur in the nonlinear constant power loads, since this is the most challenging case. The costs of the power injection of the DGUs are different for all, except for a DGU 3 of microgrid 1 and DGU 1 of microgrid 2, which have identical costs.

Figure 4 shows the power injection of the DGUs of both microgrids. When load steps occur, the power injections of both microgrids vary. Note that the load steps occur only at one microgrid, but the power injections of DGUs of both microgrids change. This is due to the price forming mechanism (15b) and the electric interconnection of the microgrids. When a load step occurs, there is an power imbalance in the network and both grid-forming DGus have to inject power. Then, the price in both microgrids raises and the power injections follow. Note, however, that at steady state, the power injection of the both DGUs with same costs are identical, achieving thus global optimality.

The price of both microgrids over the simulation time is shown in Figure 5. We see that the price varies when load steps occur, as explained above. At steady state, both prices are equal, showing again global optimality. The variation of the prices depends on where the disturbance occurs and which grid-forming DGU is primarily compensating the load. Furthermore, we see that the power injection of the grid-following DGUs in Figure 4 varies accordingly to the prices.



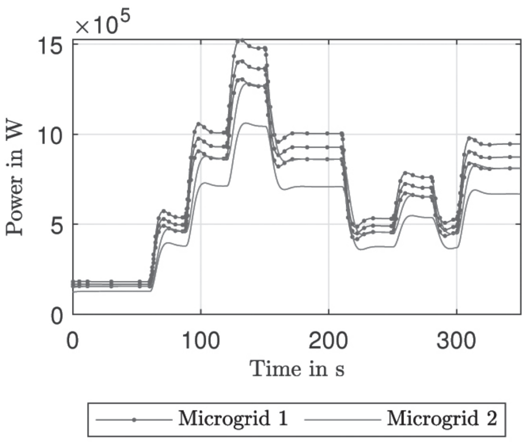


Fig. 4. Power injected by the DGUs of both microgrids over time

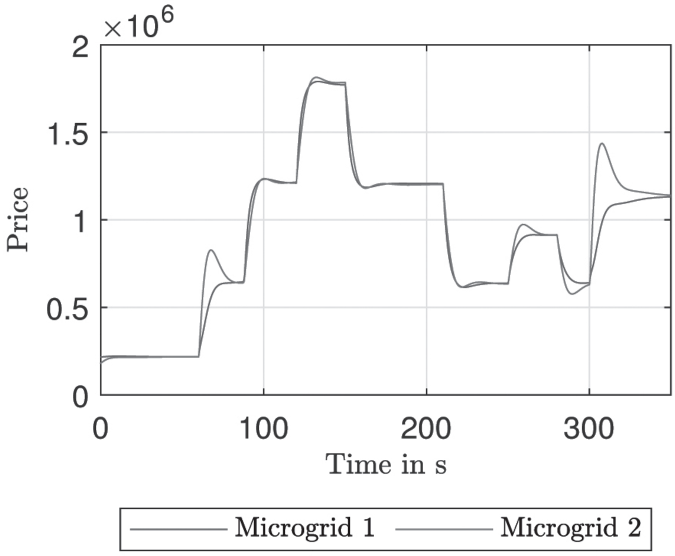


Fig. 5. Local price of both microgrids

VI. Conclusion

In this paper, we introduce the novel economic ports for networked AC microgrids. We show, that the system composed interconnected microgrids is stable and that global optimality is obtained which a special interconnection scheme for the local prices. Further work comprises the analysis of the dissipativity properties of the novel economic ports in order to achieve a modular stability analysis.

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